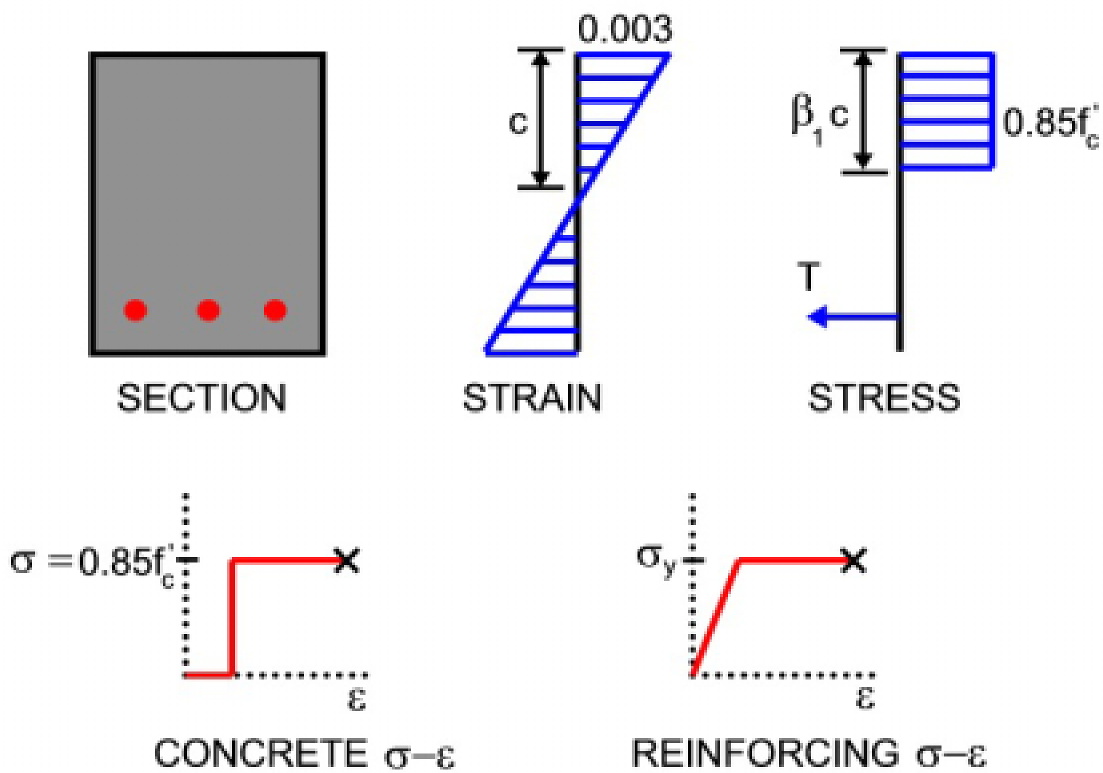
MOMENT CURVATURE ANALYSIS

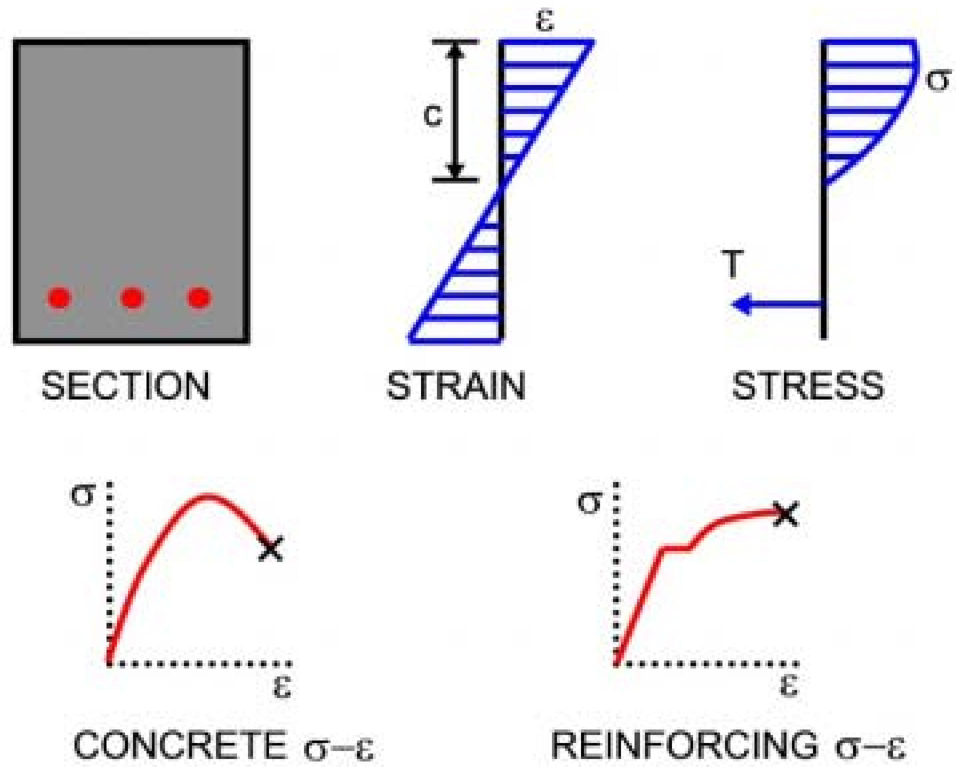
- Reinforced concrete design calculations normally assume a simple material model for the concrete and reinforcement to determine the moment capacity of a section. The Whitney stress block for concrete along with an elasto-plastic reinforcing steel behavior is the most widely used material model in American codes.

WHITNEY STRESS BLOCK



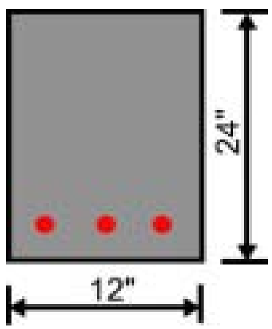
- The actual material behavior is nonlinear and can be described by idealized stress-strain models. Caltrans Seismic Design Criteria uses the Park complex strain hardening model for reinforcing steel behavior and Mander's confined and unconfined models for concrete behavior.

GENERAL STRESS BLOCK



- Moment curvature analysis is a method to accurately determine the load-deformation behavior of a concrete section using nonlinear material stress-strain relationships. For a given axial load there exists an extreme compression fiber strain and a section curvature ($\phi = \varepsilon / c$ in radians/length) at which the nonlinear stress distribution is in equilibrium with the applied axial load. A unique bending moment can be calculated at this section curvature from the stress distribution. The extreme concrete compression strain and section curvature can be iterated until a range of moment-curvature values are obtained.

- EXAMPLE 1: UNCONFINED CONCRETE SECTION



12" wide x 24" deep reinforced concrete beam
3 #9 rebars at d = 21"
axial load P = 100 kips

f'_c = 5200 psi
f_y = 60000 psi

  - Calculate moment capacity and curvature using Whitney stress block

    $\Sigma F = 0 \Rightarrow 100000 = 0.85 \times 5200 \times 12 \times a - 60000 \times 3$

    $a = 5.279$

    $\Sigma M \Rightarrow [0.85 \times 5200 \times 12 \times 5.279(12 - 5.279/2) + 60000 \times 3(21 - 12)] / 12000 = 353.4$ k-ft

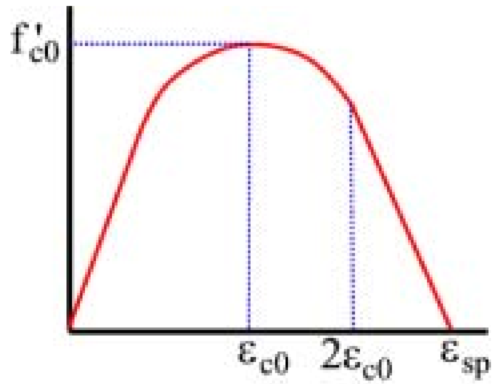
    $\beta_1 = 0.85 - (5.2 - 4)0.05 = 0.79$

    $\phi = 0.003 \times 0.79 / 5.279 = 0.000449$ rad/in

- o Calculate moment-curvature using Mander unconfined concrete model
  - Ultimate curvature is obtained when concrete reaches spalling strain

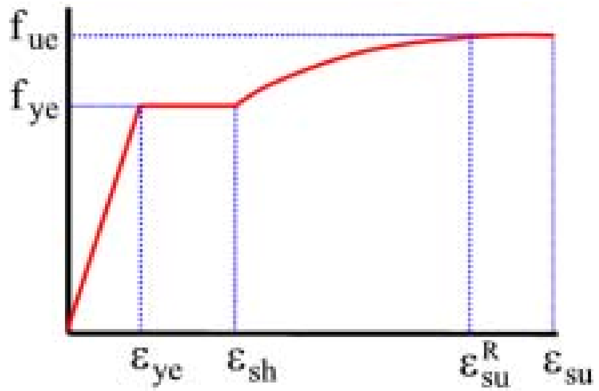
<u>Mander unconfined concrete model</u>

Unconfined strain ε_{c0} = 0.002
Spalling strain ε_{sp} = 0.005
Unconfined stress f'_{c0} = 5200 psi
Mod of elasticity E = 4110328 psi



<u>Park reinforcing model (complex strain hardening)</u>

yield strain ε_{ye} = 0.0023
yield stress f_{ye} = 68000 psi
hardening strain ε_{sh} = 0.0125
ultimate strain ε_{su}^{R} = 0.09
ultimate stress f_{ue} = 95000 psi



- Calculate moment-curvature at extreme fiber compressive strain $\varepsilon = 0.003$

ΣF = 0. at c = 6.55" (from program iteration)
curvature $\phi = 0.003 / 6.55 = 0.00458$ radians/inch

strain in reinforcing = 0.003 - 21 x .003 / 6.55 = -.006692
stress = -68000 psi
T = -68000 x 3 = -204000 lbs at y = 21

strain in concrete varies from 0. to 0.003 -- stress at 0.003 from Mander model:

$$x = \frac{strain}{\varepsilon_{c0}} = \frac{0.003}{0.002} = 1.5$$

$$E_{sec} = \frac{f'_{c0}}{\varepsilon_{c0}} = \frac{5200}{0.002} = 2600000$$

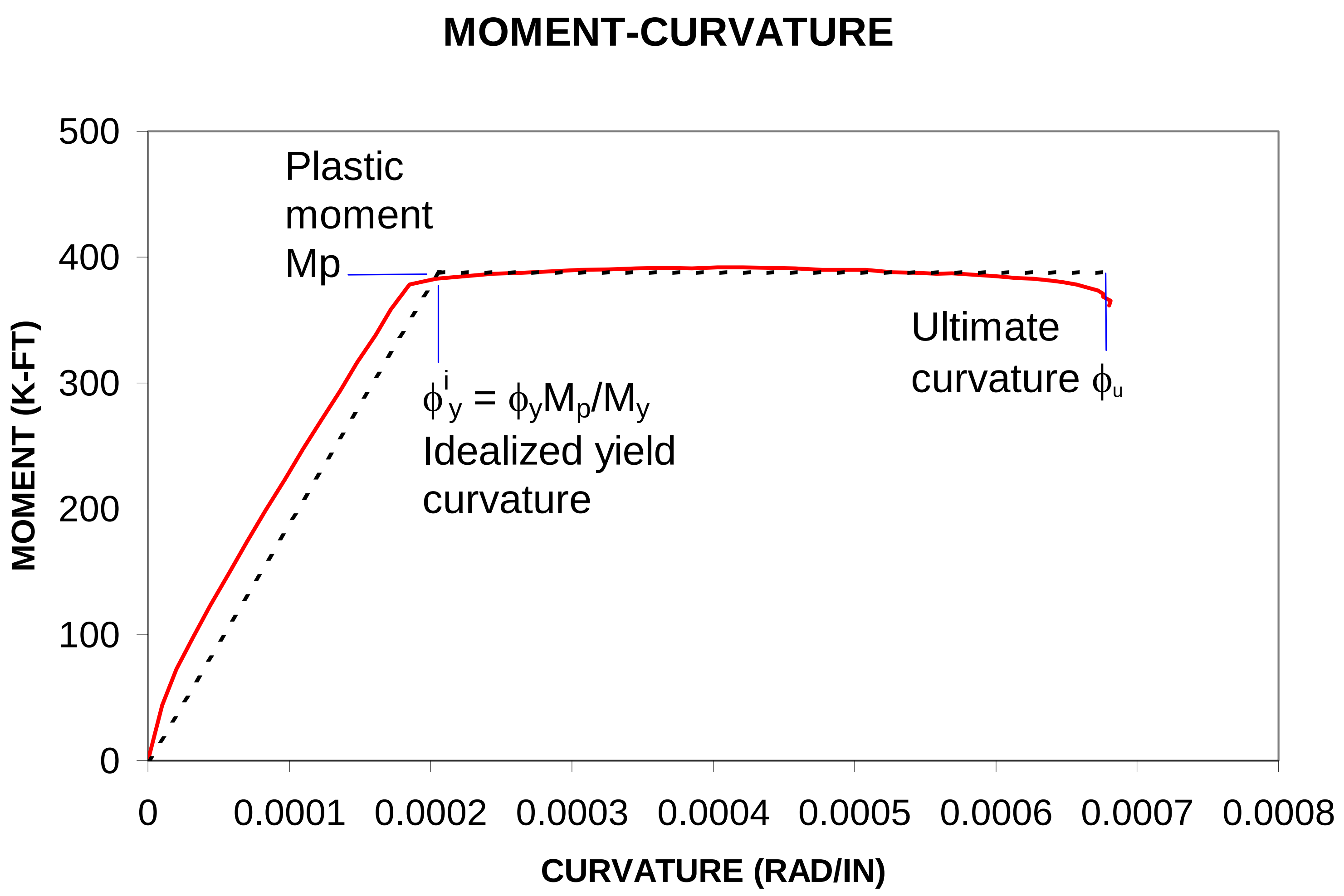
$$r = \frac{E_c}{E_c - E_{sec}} = \frac{4110328}{4110328 - 2900000} = 2.72$$

$$stress = \frac{f'_{c0} x r}{r - 1 + x^r} = \frac{5200(1.5)(2.72)}{2.72 - 1 + 1.5^{2.72}} = 4483 \text{ psi}$$

integration yields C = 305363 lbs at y = 2.653

Resultant axial load = 305 - 204 = 101 kips ≅ 100 kips   Okay
eccentricity = 12 - (305.363 x 2.653 - 204 x 21) / 101.363 = 46.27 in
Moment = 101.363 x 46.27 / 12 = 390.9 k-ft

- Calculate moment-curvature for a range of strain values to get figure below



- Caltrans Seismic Design Criteria parameters

  - Cracked moment of inertia, I_{cr}, may be determined from curvature at first yield of reinforcing.

$$I_{cr} = \frac{M_y}{E\phi_y} = \frac{382.7(12)}{4110.3(0.000204)} = 5480\,\text{in}^4$$

  - Plastic moment, M_p, may be determined from average moment after first yield.

M_p = 387.4 k-ft (compares to 353.4 k-ft for Whitney stress block)

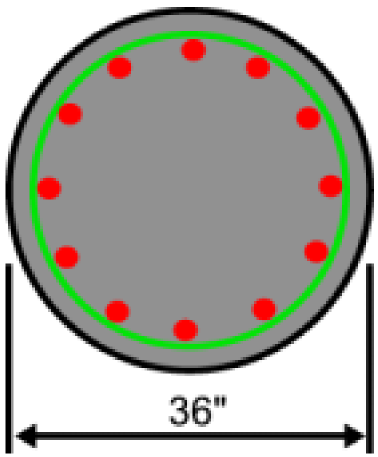
  - Idealized yield curvature is the curvature at the elastic-plastic transition point

$$\phi_y^i = \phi_y \frac{M_p}{M_y} = 0.000204\left(\frac{387.4}{382.7}\right) = 0.000206$$

  - Ultimate curvature at point when failure strain of concrete or reinforcing is reached

ϕ_u = 0.00068 at concrete spalling strain of 0.005

- EXAMPLE 2: CONFINED CONCRETE SECTION

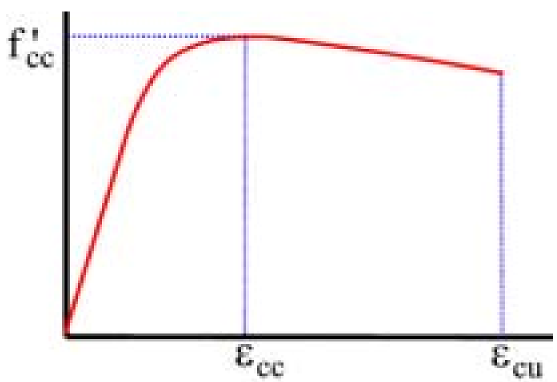


24" diameter reinforced concrete column
12 #9 rebars at r = 14.75"
#4 spiral (d_s = 0.5") at pitch s_t = 3"
spiral diameter to centerline D_c = 31.5 inches
axial load P = 1000 kips

- Calculate moment-curvature using Mander confined concrete model
  - Concrete spalling failure is modeled outside of confinement reinforcement
  - Ultimate curvature is limited by confinement reinforcement failure or longitudinal reinforcement failure

Mander confined concrete model

Unconfined strain ε_{c0} = 0.002
Confined strain = ε_{cc}
Spalling strain ε_{sp} = 0.005
Unconfined stress f'_{c0} = 5200 psi
Confined stress = f'_{cc}
Mod of elasticity E = 4110328 psi



Park reinforcing model (complex strain hardening)

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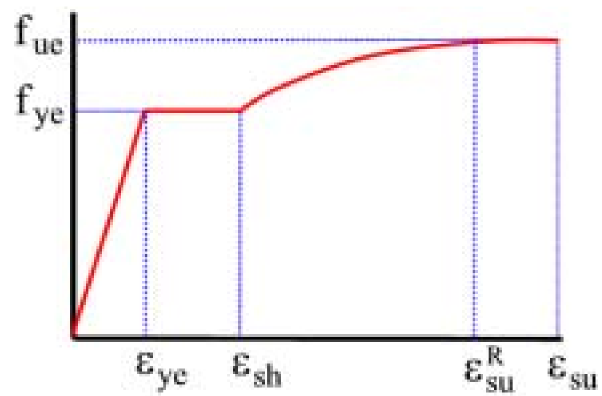


- Confining pressure f_{lp} is calculated based on confinement reinforcement

  - The confinement stress calculations for spiral reinforcement are shown below

$A_{core} = \pi (D_c)^2 / 4 = \pi (31.5)^2 / 4 = 779.3$
$A_{cc} = A_{core} - A_{reinf} = 779.3 - 12 = 767.3$
$d_e = D_c - (s_t - d_s) / 2 = 31.5 - (3 - 0.5) / 2 = 30.25$
$A_e = \pi D_c \times d_e / 4 = \pi (31.5) \times 30.25 / 4 = 748.4$

$k_e = A_e / A_{cc} = 748.4 / 767.3 = 0.975$
$\rho_s = \pi (d_s)^2 / (D_c \times s_t) = \pi (0.5)^2 / (31.5 \times 3) = 0.00831$
$f_{lp} = k_e \times \rho_s \times f_{yh} / 2 = 0.975 \times 0.00831 \times 68000 / 2 = 275$ psi

- Confined stress and strain from Mander's formulas

$$f'_{cc} = f'_{co}\left[2.254\sqrt{1+\frac{7.94 f_{lp}}{f_{co}}} - \frac{2 f_{lp}}{f_{co}} - 1.254\right]$$

$$f'_{cc} = 5200\left[2.254\sqrt{1+\frac{7.95(275)}{5200}} - \frac{2(275)}{5200} - 1.254\right] = 6899$$

$$e_{cc} = e_{co}\left[5\left(\frac{f'_{cc}}{f'_{co}} - 1\right) + 1\right] = 0.002\left[5\left(\frac{6899}{5200} - 1\right) + 1\right] = 0.00527$$

- Maximum strain from Priestley's formula

$\varepsilon_{cu} = 0.004 + 1.4 \times \rho_s \times f_{yh} \times e_{su} / f'_{cc}$

$\varepsilon_{cu} = 0.004 + 1.4 \times 0.00831 \times 680000 \times 0.09 / 6899 = 0.01432$

- Alternately, the ultimate strain can be determined from complicated strain energy balance formulas.

STRUCTSOURCE

- Computer tools available for moment-curvature analysis

  - XSECTION (Caltrans)

    Versatile fiber model good for any section. Park material model for reinforcing steel. Mander confined and unconfined models for concrete. DOS program uses batch input. English units only.

  - CONSEC (my program)

    Integration for any combination of concrete rectangle and circular segments, reinforcing line or arc and structural steel bar or pipe. Park and bilinear model for reinforcing steel. Bilinear, simple and Mander confined and unconfined models for concrete. Windows program.

  - UCFyber (ZEVENT)

    Extremely versatile fiber model good for any section. Various material models for reinforcing steel and concrete. Commercial windows program.

  - SEQMC (SEQAD)

    Priestley's program for circular or rectangular section analysis only. Various material models for reinforcing steel and concrete. Shareware windows program.

- XSECTION input file

```
xSECTION
VER._2.40,_MAR-14-99
LICENSE    (choices: LIMITED/UNLIMITED)
LIMITED
ENTITY     (choices: GOVERNMENT/CONSULTANT)
CONSULTANT
NAME_OF_FIRM
HOLMES&NARVER
BRIDGE_NAME
RIVERSIDE_AVE_OC
BRIDGE_NUMBER
54-0623
JOB_TITLE
example 2 column analysis
*****************************************************
Subsection definition is supported by coordinates
bending parallel to x-axis (horiz.)
local x- and y- axes parallel to global X- and Y-
Units are Kips and inches
```

HEADER MUST FOLLOW EXACT FORMAT PROVIDED BY CALTRANS FOR PROGRAM TO WORK

STRUCTSOURCE

```
*****************************************************
concrete material model section
type 1 concrete is mander confined
type 2 concrete is mander unconfined
*****************************************************
CONC_TYPES_START
NUMBER_OF_TYPES 2
TYPE_NUMBER 1   MODEL mander
    CONFINED_SUBSECTION_SHAPE  circular
    CONFINED_SUBSECTION_DIAM 31.5
    CONF_TYPE spiral
        CONF_STEEL_TYPE 1  CONF_BAR_AREA 0.20  CONF_BAR_DIAM 0.50
        CONF_BAR_SPACING  3.0
        MAIN_BAR_TOTAL  12   MAIN_BAR_AREA  1.00
    STRAIN_e0  0.002    STRAIN_eu 0.005   ULT_STRAIN_FACT  1.00
    STRESS_f0  5.2   STRESS_fu  2.5
    UNIT_WEIGHT_FACT  0.96
TYPE_NUMBER 2   MODEL unconfined_mander
    STRAIN_e0  0.002    STRAIN_eu 0.005   ULT_STRAIN_FACT  1.0
    STRESS_f0  5.2   STRESS_fu  2.5
    UNIT_WEIGHT_FACT  0.96
CONC_TYPES_END
*****************************************************
reinforcing steel material model
Type 1 is park model for ASTM A706 Grade 60 #9 bars
*****************************************************
STEEL_TYPES_START
NUMBER_OF_TYPES 1
TYPE_NUMBER 1   MODEL park
YIELD_STRAIN 0.0023  HARDEN_STRAIN 0.0125   ULT_STRAIN 0.09
YIELD_STRESS 68.0    ULT_STRESS 95.0
MODULUS  29000.0
STEEL_TYPES_END
```

CONCRETE MATERIAL MODEL SECTION

REINFORCING MATERIAL MODEL SECTION

STRUCTSOURCE

```
*****************************************************
define concrete fibers
subsection 1 is confined area
subsection 2 is unconfined area
********************************************************
SUBSECTION_START
NUMBER_OF_SUBSECTIONS  2
SUBSECTION_NUMBER 1
    SHAPE arc_strip
    CENTER_GLOBAL_X_Y 0 0 START_ANGLE 0 DURATION_CCW 360
    RADIUS_OUTER 15.75 RADIUS_INNER 0
    NUMBER_OF_FIBERS_RADIAL   10      NUMBER_OF_FIBERS_ANGULAR   36
    CONC_TYPE  1
    MIRROR_4_WAYS   no
SUBSECTION_NUMBER 2
    SHAPE arc_strip
    CENTER_GLOBAL_X_Y 0 0 START_ANGLE 0 DURATION_CCW 360
    RADIUS_OUTER 18 RADIUS_INNER 15.75
    NUMBER_OF_FIBERS_RADIAL   2      NUMBER_OF_FIBERS_ANGULAR   36
    CONC_TYPE  2
    MIRROR_4_WAYS   no
SUBSECTION_END
********************************************************
define reinforcing fibers
********************************************************
REBAR_LAYOUT_START
NUMBER_OF_REBAR_GROUPS  1
GROUP_NUMBER 1
    LAYOUT_SHAPE  circular
    NUMBER_OF_REBARS  12   AREA_OF_EACH_BAR  1.00  STEEL_TYPE      1
    CENTER_GLOBAL_COORD_X_Y 0 0 START_ANGLE 0. DURATION_CCW 360
    RADIUS 14.75
    MIRROR_4_WAYS   no
REBAR_LAYOUT_END
```

CONCRETE GEOMETRY SECTION

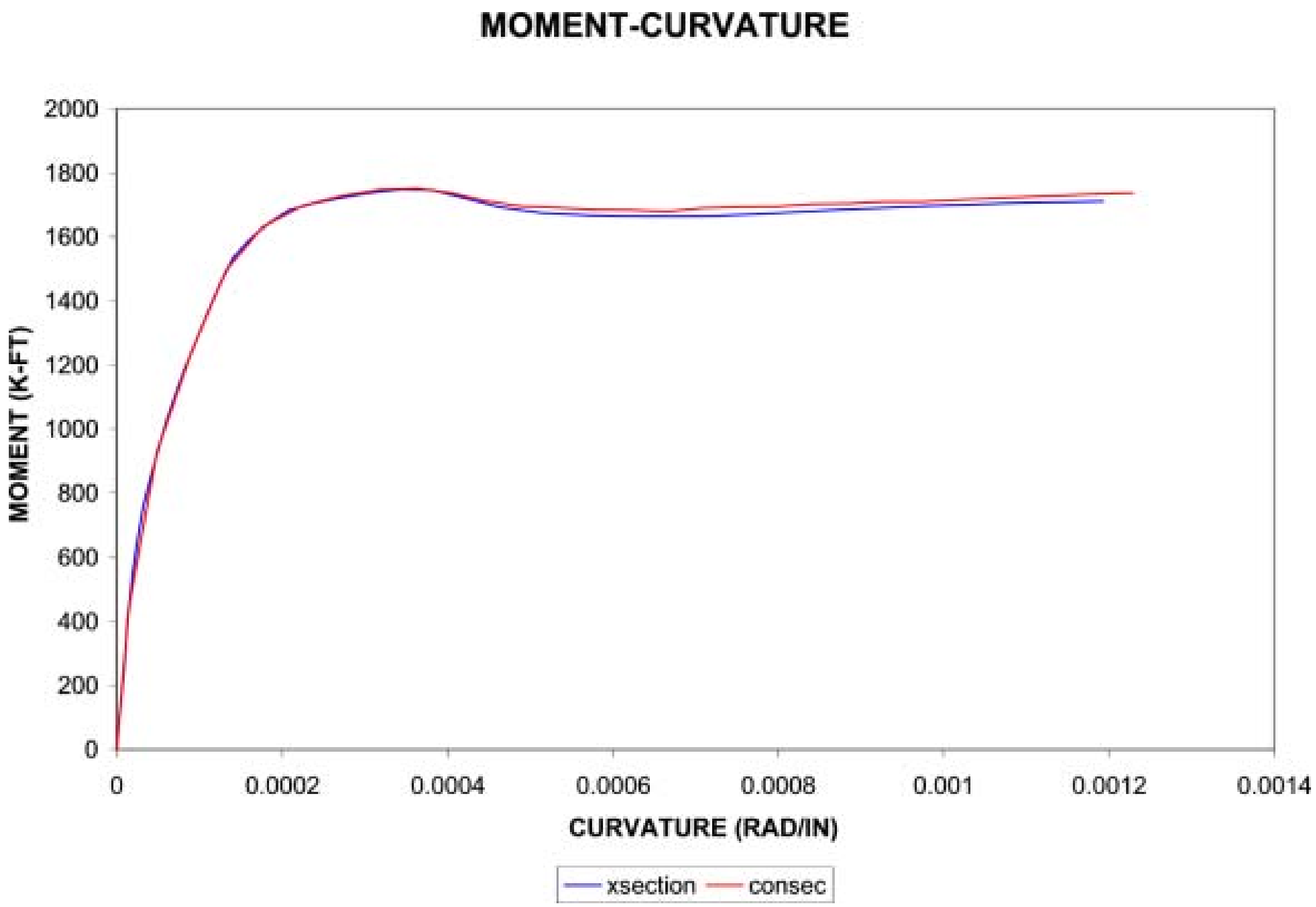
REINFORCING GEOMETRY SECTION

LOADS SECTION

```
********************************************************
AXIAL_LOAD
    LOAD_VALUE   1000
    CENTER_OF_LOAD_APPLICATION_GLOBAL_X_Y    0    0
********************************************************
analysis control parameters
Let the cover concrete fail but stop at core concrete failure or the
first longitudinal rebar failure.
To control the initial guess of the Neutral Axis a factor is defined which
varies from 0.01 to 0.99 as shown below. This is used if there is
instablity in the moment-curvature curve.
********************************************************
ANALYSIS_CONTROL
    STOP_DUE_FIRST_CONC_FAILURE   no
    STOP_DUE_FIRST_REBAR_FAILURE   yes
    BENDING_AXIS_CCW_ROTATION_DEGREES    0
    NEUTRAL_AXIS_PROXIMITY_TO_COMPRESSION_EDGE      0.99
    CONVERGENCE_TOLERANCE 0.001
*********************************************************
RESULTS_REQUESTED
    MOMENT_AT_GLOBAL_X_Y    0    0
    CONC_FIBER_INFO_OUTPUT      yes
    REBAR_FIBER_INFO_OUTPUT     yes
*********************************************************
```

ANALYSIS AND OUTPUT SECTION

- MOMENT-CURVATURE PLOT



- REFERENCES

1. Priestley, Seible, Calvi, "Seismic Design and Retrofit of Bridges", John Wiley & Sons,1996.

2. Seyed, Lee, "Material Modeling in Section and Column Analysis", Caltrans, April 14, 1994

3. Mander, Priestley, Park, "Theoretical Stress-Strain Model for Confined Concrete, *J. Struct. Eng.*, ASCE, 1988, 114(8), p. 1804-1849.

STRUCTSOURCE